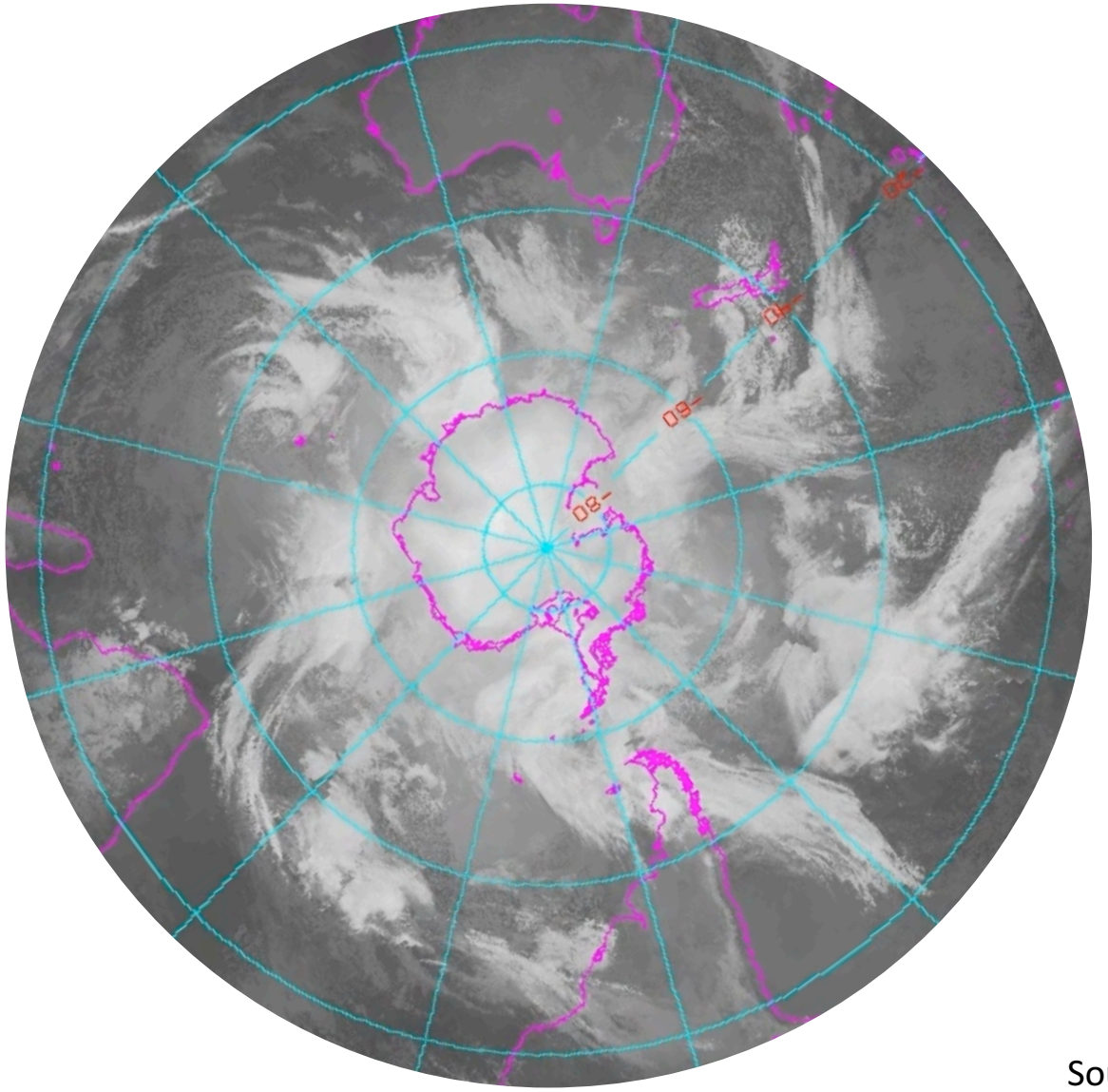
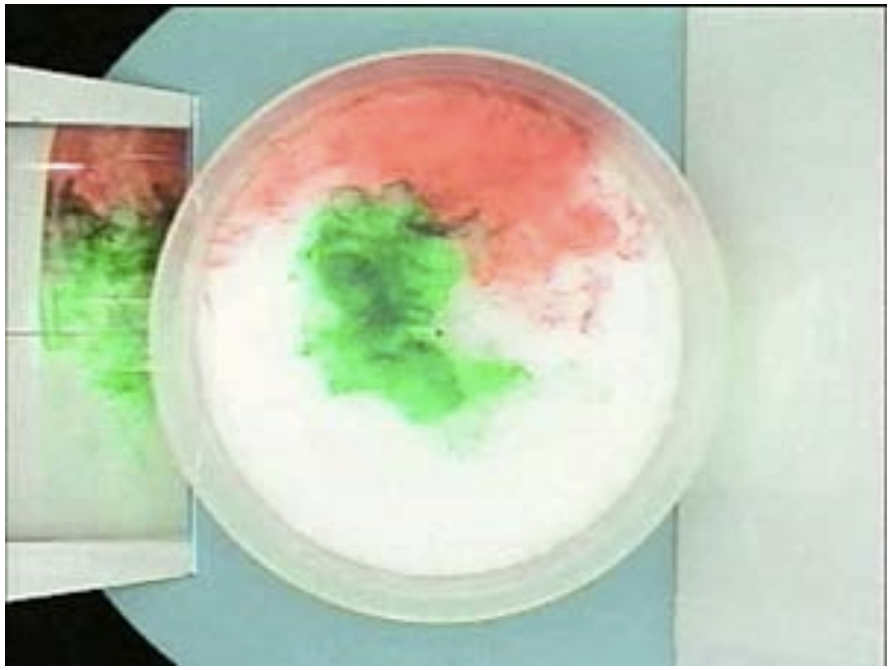
Why should we care about rotating flows?

Earth rotates!



Source: Bureau of Meteorology

Rotating and non-rotating flows behave qualitatively differently!



Why?

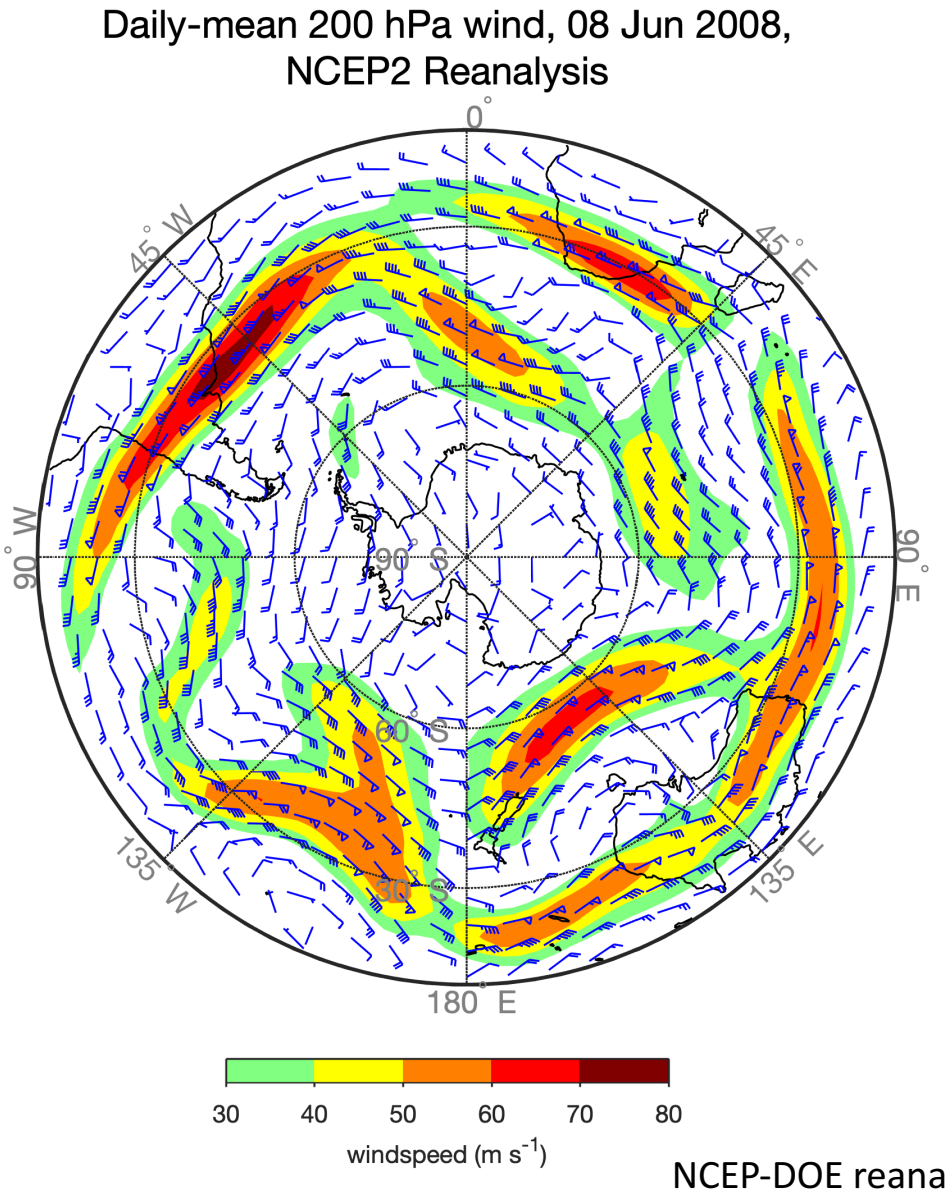
Source: Weather in a Tank, MIT

Two *amazing* facts

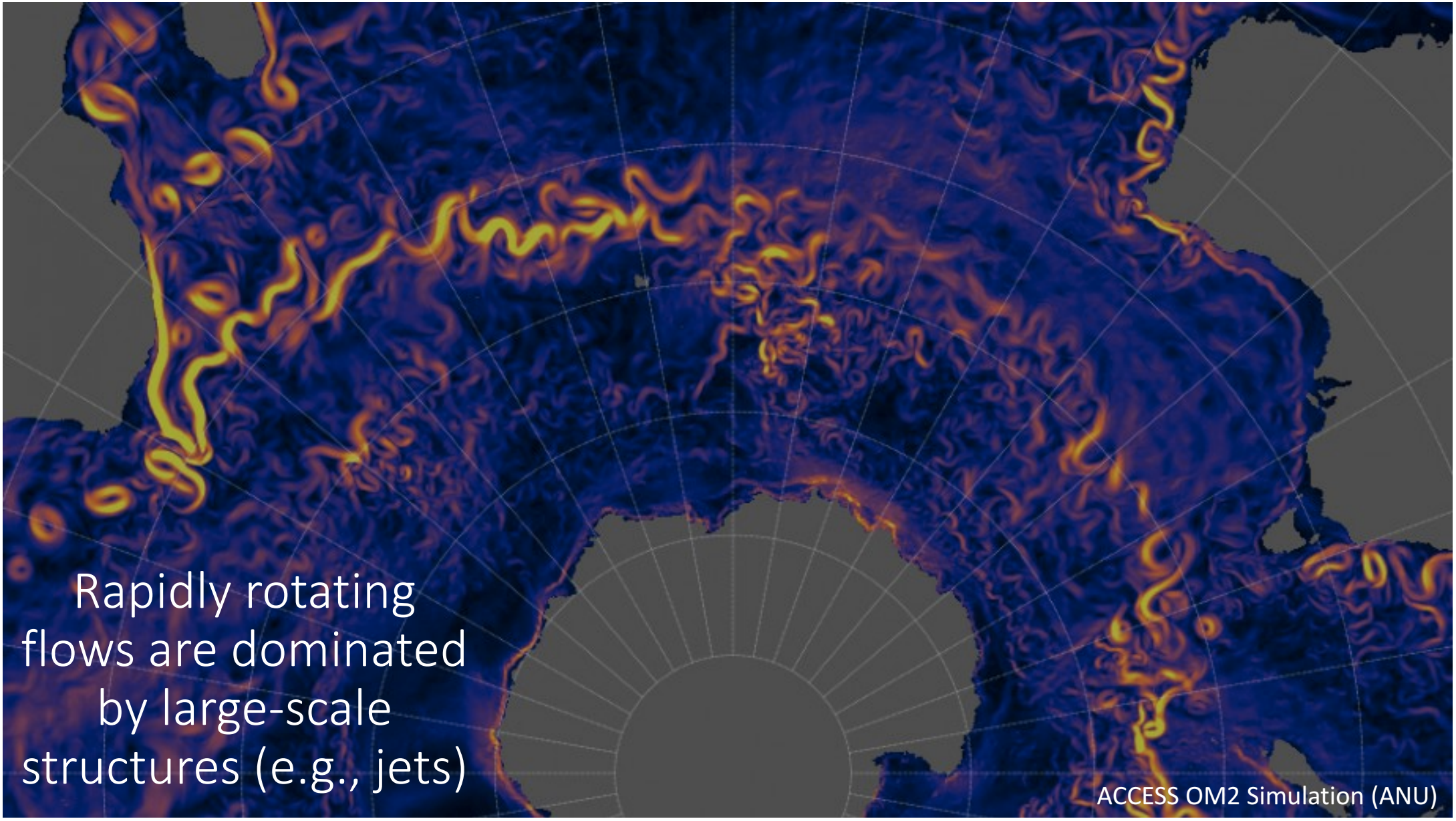
1. **Rapidly rotating flows are “quasi-two-dimensional”**

2. **In 2D turbulence, energy builds up at the largest scales**

Rapidly rotating flows are dominated by large-scale structures (e.g., jets)

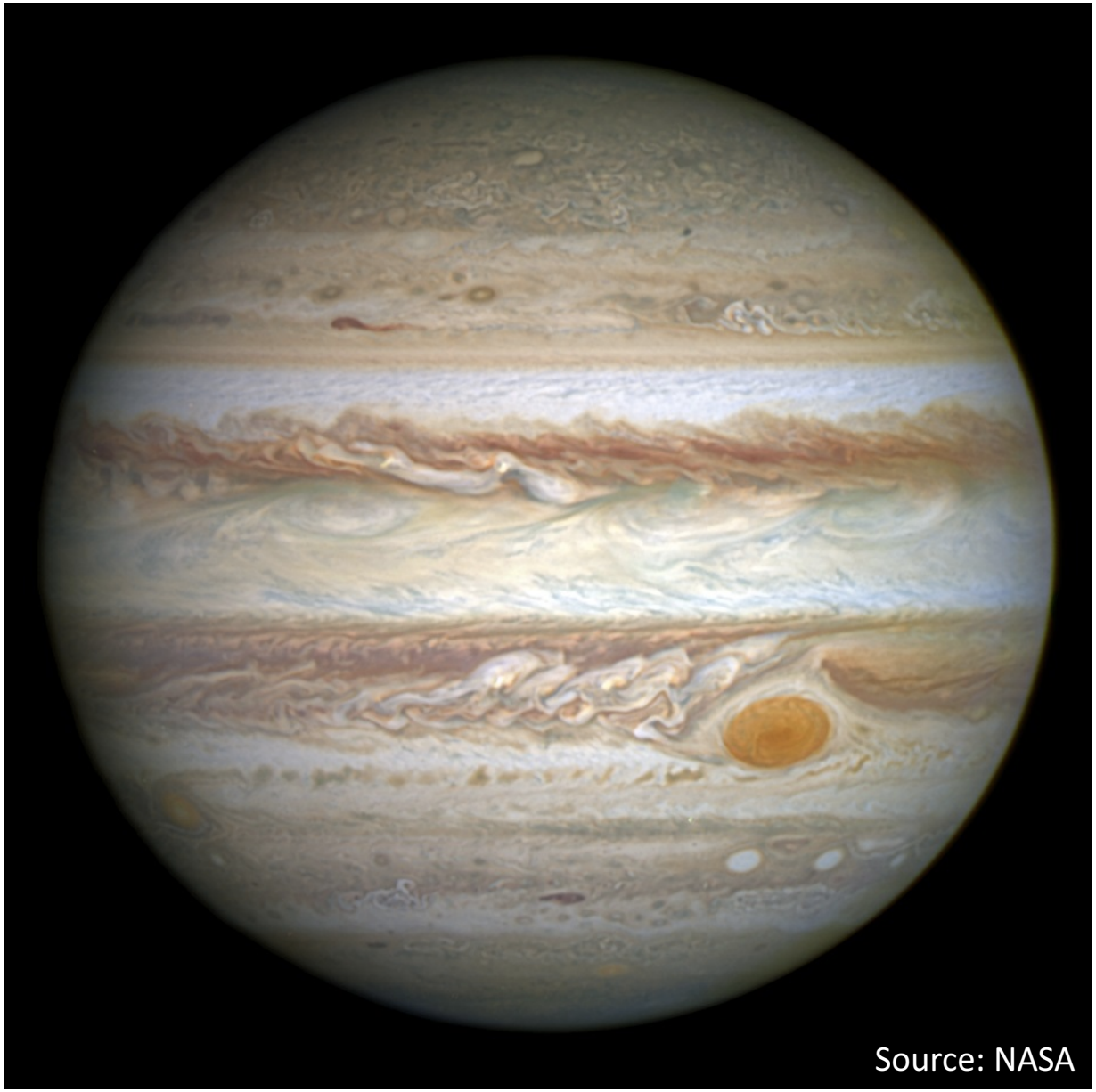


NCEP-DOE reanalysis


Rapidly rotating flows are dominated by large-scale structures (e.g., jets)
ACCESS OM2 Simulation (ANU)

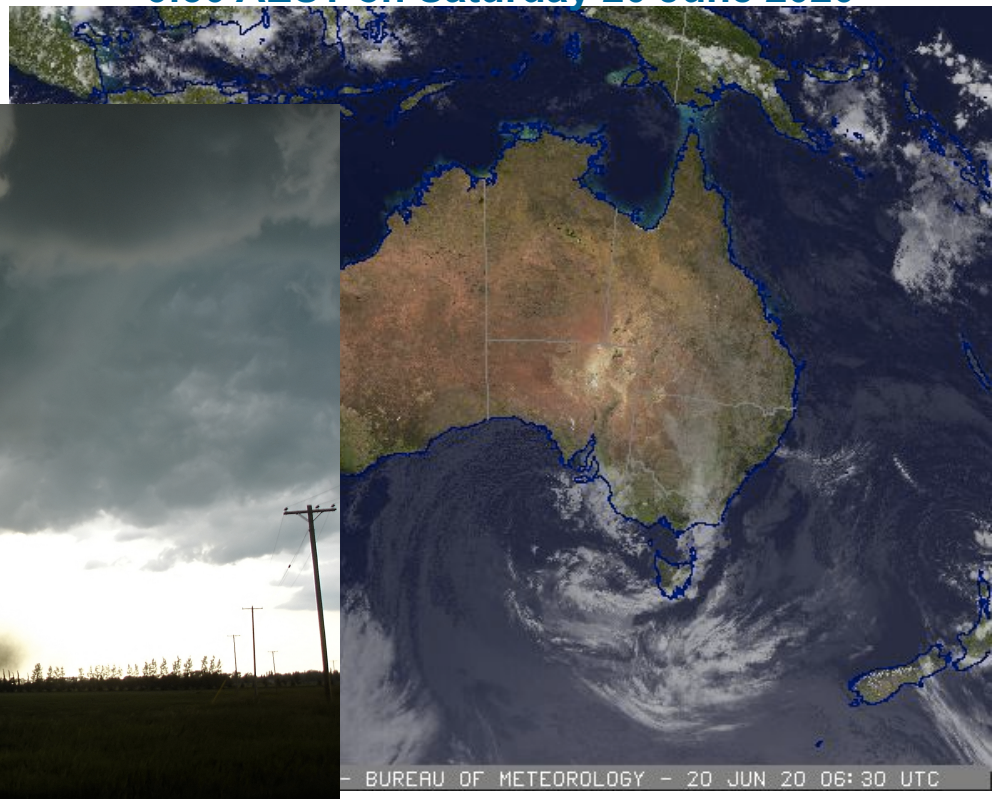
Rapidly rotating flows produce long-lived coherent structures

Friction plays a very different role in 2D versus 3D turbulence



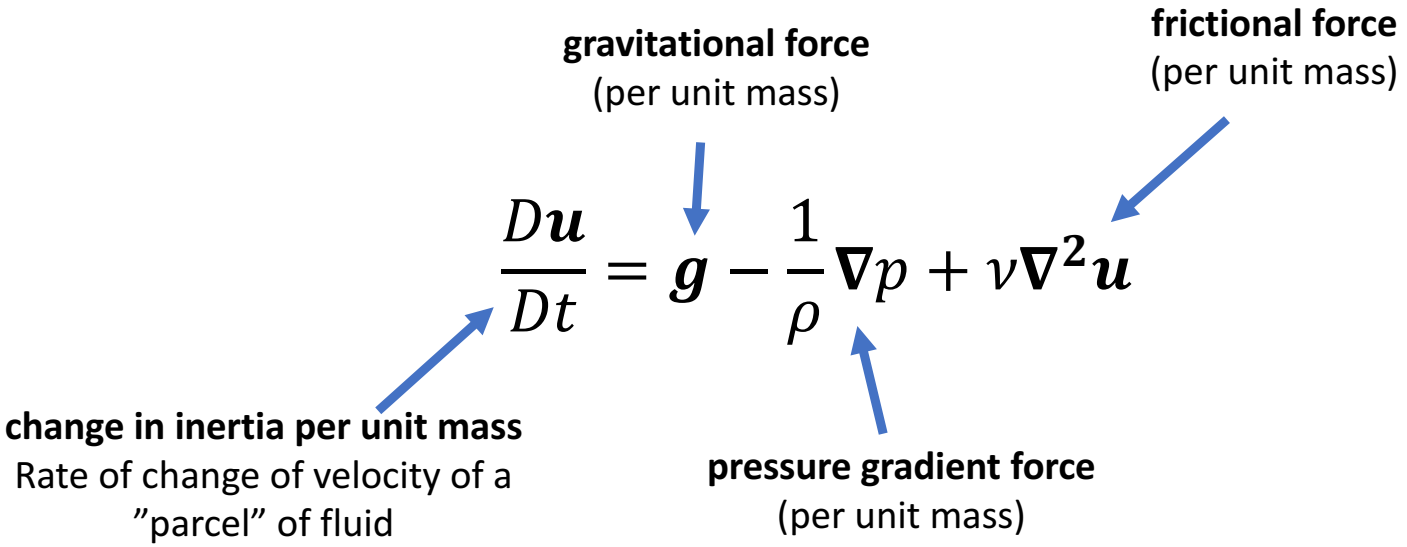
When is the Earth's rotation important?

6:30 AEST on Saturday 20 June 2020



The momentum equation

The momentum equation applicable to the atmosphere and oceans (the Navier-Stokes equation):



Momentum equation in rotating coordinates

- ***Centrifugal force:*** $-\boldsymbol{\Omega}\times\boldsymbol{\Omega}\times\boldsymbol{r}$
  - Acts on all objects, stationary or moving
  - Directed away from the Earth's rotation axis
- ***Coriolis force:*** $-2\boldsymbol{\Omega}\times\boldsymbol{u}$
  - Acts on objects that are moving (relative to the Earth's surface)
  - Acts at right-angles to the direction of motion
  - Does no work

The tangent plane approximation

Useful form of the momentum equations on the Earth

$$\frac{Du}{Dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{Dv}{Dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$f = 2\Omega \sin\phi$

$$\frac{\partial p}{\partial z} = -\rho g$$

gravity + centrifugal force

Non dimensionalised equations

$$\frac{D\hat{u}}{D\hat{t}} = \frac{\hat{v}}{Ro} - \frac{1}{\rho U^2}\frac{\partial p}{\partial \hat{x}} + \frac{1}{ReA^2}\frac{\partial^2 \hat{u}}{\partial \hat{z}^2}$$

$$Ro = \frac{U}{fL}$$

$$Re = \frac{UL}{\nu}$$

$$A = \frac{H}{L}$$

$f \approx 1\times10^{-4}$ s^{-1}

$\nu \approx 1\times10^{-5}$ m^2 s^{-1}

- **Which terms are important in a bathtub?**
- **Which terms are important in a tornado?**
- **Which terms are important in a midlatitude cyclone?**

High Reynolds number, low Rossby number:

$$\frac{D\hat{u}}{D\hat{t}} = \frac{\hat{v}}{Ro} - \frac{1}{\rho U^2}\frac{\partial p}{\partial \hat{x}} + \frac{1}{ReA^2}\frac{\partial^2 \hat{u}}{\partial \hat{z}^2}$$

Geostrophic balance